$$x_1(k) = q(k+2)$$
$$x_2(k) = q(k+1)$$
$$x_3(k) = q(k)$$

$$x_3(k+1) = x_2(k)$$
$$x_2(k+1) = x_1(k)$$
$$x_1(k+1) = q(k+3) = e(k) - a_1 x_1(k) - a_2 x_2(k) - a_3 x_3(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} -a_1 & -a_2 & -a_3 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} e(k)$$

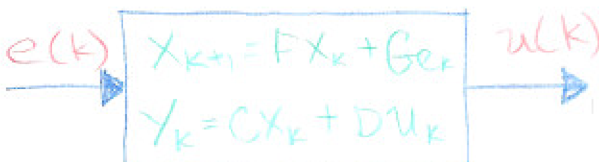
(2) $\Rightarrow$ $u(z) = b_0 z^3 Q(z) + b_1 z^2 Q(z) + b_2 z Q(z) + b_3 Q(z)$

$$u(k) = b_0 \left[e(k) - a_1 x_1(k) - a_2 x_2(k) - a_3 x_3(k) \right] + b_2 x_2(k) + b_3 x_3(k)$$

$$u(k) = Cx(k) + De(k)$$

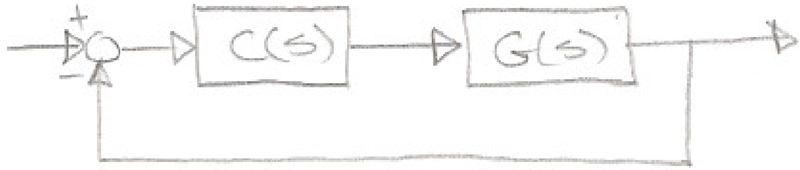
$$C = \left[b_1 - a_1 b_0,\ b_2 - a_2 b_0,\ b_3 - a_3 b_0 \right]$$

$$D = b_0$$



we have generally 2 methods for computer based control.

Method 1 (continuous time)

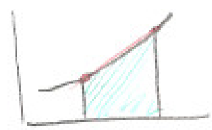


1. Design continouous time controller $C(s)$
2. Digitize the continous time controller $C(s)$ to obtain a set of difference equations to be implemented in the computer.

There are different methods to obtain the digitized controller

Euler Method.

$$\dot{x} \cong \frac{x(k+1) - x(k)}{T}$$



Ex:

$$C(s) = k\frac{s+a}{s+b} = \frac{u(s)}{E(s)}$$

$$k(s+a)E(s) = (s+b)u(s)$$

$$\mathcal{L}^{-1} \Rightarrow \quad k\dot{e} + kae = \dot{u} + bu$$

$$k\left(\frac{e(k+1) - e(k)}{T}\right) + kae(k) =$$

$$= \frac{u(k+1) - u(k)}{T} + bu(k)$$

$$u(k+1) = (1 - bT)u(k) + Tk\left(\frac{e(k+1) - e(k)}{T} + ae(k)\right)$$

z trans $\Rightarrow z\,U(z) = (1-bT)\,U(z) + TK\left(\frac{z\,e(z) - e(z)}{T} + a\,e(z)\right.$

$$(z - 1 + bT)\,U(z) = Kz\,e(z) - K\,e(z) + aTK\,e(z)$$

$$\boxed{\frac{U(z)}{e(z)} = \frac{Kz - K + aTK}{z - 1 + bT} = D(z)}$$

Matched Pole Zero Method.

$C(s) \rightsquigarrow D(s)$ by mapping the poles and zeros according to the relationship: $z = e^{sT}$

sampling time

Method 2 (Direct Digital Control Design)

Find the exact G(z)

$$G(z) = (1 - z^{-1})\, Z\left\{\frac{G(s)}{s}\right\}$$

note: $G(z) = n_1 / d_1$

$C(s) = n_2 / d_2$

matlab: $[n_1, d_1] = c2d(n_2, d_2, T, \text{'ZOH'})$

Design $D(z)$ directly

Ex: $\dfrac{G(s)}{s} = \dfrac{s}{s(s+a)} = \dfrac{1}{s} + \dfrac{1}{s+a}$

$$\mathcal{L}^{-1}(G(s)) = 1(t) - e^{-at}\,1(t)$$

$$z\left\{\frac{G(s)}{s}\right\} = \frac{z}{z-1} - \frac{z}{z-e^{-at}} = \frac{z(1-e^{-at})}{(z-1)(z-e^{-at})}$$

$$G(z) = (1-z^{-1})\,z\left\{\frac{G(s)}{s}\right\} = \frac{1-e^{-at}}{z-e^{-at}}$$